Exercise 44

Evaluate the integral.

$$\int_{-2}^{2} f(x)\,dx \quad \text{where } f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4 - x^2 & \text{if } 0 < x \le 2 \end{cases}$$

Solution

Split up the integral over the intervals on which the given function is defined. Then use the second part of the fundamental theorem of calculus to evaluate the integrals.

$$\begin{aligned} \int_{-2}^{2} f(x)\,dx &= \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx \\ &= \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4 - x^2)\,dx \\ &= (2x)\Big|_{-2}^{0} + \left(4x - \frac{x^3}{3}\right)\Big|_{0}^{2} \\ &= [2(0) - 2(-2)] + \left[4(2) - \frac{(2)^3}{3}\right] - \left[4(0) - \frac{(0)^3}{3}\right] \\ &= 4 + 8 - \frac{8}{3} \\ &= \frac{28}{3} \end{aligned}$$